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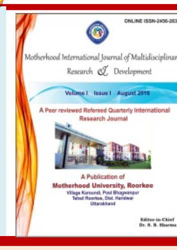

Study of an Inventory Model with Time Varying Holding cost, Exponential Decaying Demand and Constant Deterioration

Dr.Sandeep Kumar
Associate Professor
Dewan V.S. Group of Institutions
Meerut, Uttar Pradesh

Abstract

In this paper an optimal cost of inventory is considered under the exponential decaying demand and constant deterioration .Here we taken time dependent holding cost which is a linear function of time. Also the shortages are not allowed. We have so many items in our daily life, have fixed life and lost their values with time. For example foods, fruits, fashion apparels, flowers, vegetables etc. To demonstrate the model, numerical example is presented.

Introduction

In the present study the objective is to develop an inventory model for deteriorating items with time dependent holding cost and exponential decaying demand. Deteriorating rating items means that the items that become damaged or lost their marginal value through time. There are two types of deterioration of items. The first type of items that become spoiled or expired with time like, vegetables, foods, fruits, flowers, medicines etc, The second type of items that loss partially or completely their total values with time because of the of new invention of technology like items related to modern fashion, computer chips and electronic equipments , mobile phones etc..The natural life cycle of the first kind of items is low and other kind of items has a market of short life.

The main objective of the our present study is to create an inventory model for deteriorating items having time dependent holding cost, exponential declining demand rate and constant deterioration. In general the deteriorating items mean the items that become spoiled, damaged, decayed or lost of its marginal value with time. A brief and valuable review on deteriorating inventory models given by.Raafat in 1991. A valuable study on deteriorating inventory models given by Harris in 2009. Later Wilson extended Harris' model and obtain a result for economic order quantity. An inventory model developed by Chang and Dye in 1999 in which a group of customers who want to accept backlogging is the reciprocal of a linear function of the waiting time. Recently in 2000, Papachristos and S. Kouri developed an inventory model with varying demand, partially backlogged and constant deterioration rate, in which the backlogging rate depends exponentially to the waiting time.

After this many related papers presented related such type of inventory system, such as Goyal and Giri in 2001. Abad in 2001, Tenget. al. In 2002 -2003.Wang in 2002.Papachristos and Skouri in 2003. In 1990, Datta and Pal developed a model where the rate of demand of an item is dependent on the stock inventory until we have given inventory level, after that the demand rate becomes constant. In 1995, Goswami and Chaodhary improved the model of Urban developed in 1992 for deteriorating items. In 1997, Ray and Chaudhuri suited an economic order quantity inventory model with stock-dependent demand, inflation and shortages where the time is not counting for different costs and prices related to the system. In 2006, Ray, Chaudhuri and Goswami presented an inventory model with two levels of storage, a stock-dependent demand, rented and own warehouse. In 1998 Giri and Chaudhuri improved a model of Goh (developed in 1994) to consider an inventory of a deteriorating item and mentioned the case where the holding cost depends on both nonlinear time and stock. In 1957, Whitin developed an inventory model where the items related to fashion, deteriorate at the end of storage period. After that Ghare and Schrader included in a study that the use of the deteriorating items is very nearly relative to a negative exponential function of time.

They developed an economic ordering quantity model with constant deterioration rate where shortages are not allowed. In 1998, Dave and Patel presented an inventory model for decaying items with linear increasing demand without shortages. In the general study of inventory theory both the demand rate and holding cost are assumed to be constant. But in reality, it is observed that for physical goods both are time-independent.

Assumptions and Limitations

The following assumptions are made in developing the model.

1). Single item inventory system is considered only.

2). Shortages are not allowed

3).The deterministic demand rate is an exponential decaying linear function of time

4).The deterioration is constant.

5). Lead time is considered as zero.

6). Finite time horizon inventory system.

Notations

To develop the model, the following notations have been used.

Q(t) = Inventory at time t, $0 \le t \le T$

E(t) = The exponential demand rate where E(t) = M $e^{-\lambda t}$, M>0, λ >0, M>> λ , M stands for the constant demand rate and λ is the change rate in demand.

P = Fixed ordering cost per order.

T = Length of order cycle.

$\theta(t)$ = Constant deterioration cost of an item where $\theta(t) = \theta$ ($0 < \theta < 1$) and $0 < \lambda < \theta$.

g(t) = Time dependent holding cost per unit time where g(t) = g + at, g>0 and a>0.

Q_0 = Economic order quantity.

C = Cost of each deteriorated items.

TC = Total cost per unit time.

TC* = Minimum total cost per unit time

T* = Optimal length of cycle.

Q_0* = Optimal economic order quantity.

Formulation of the Model

Since the inventory reduces due to both demand and deterioration during the cycle [0,T], therefore, the generated differential equation for the presented model

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -E(t), \quad 0 \le t \le T \tag{1}$$

Where $\theta(t) = \theta$ and $E(t) = M\,e^{-\lambda t}$,

After solving (1) with boundary condition Q(t) = 0, we have

$$Q(t) = \frac{M}{(\theta-\lambda)}[e^{(\theta-\lambda)T-\theta t} - e^{-\lambda t}], \quad 0 \le t \le T \tag{2}$$

Now use the boundary condition Q(0) = Q_0, we have

$$Q_0 = \frac{M}{(\theta-\lambda)}[e^{(\theta-\lambda)T} - 1] \tag{3}$$

Here Q_0 is initial order quantity.

Now we calculate the different inventory costs.

(1). Ordering Cost = P

(2).Deterioration Cost

The total demand during the period [0,T] is

$$\int_0^T E(t)dt = \int_0^T Me^{-\lambda t}dt = \frac{M}{\lambda}[1 - e^{-\lambda T}]$$

The number of deterioration unit = $Q_0 - \int_0^T E(t)dt = = \frac{M}{(\theta-\lambda)}[e^{(\theta-\lambda)T} - 1] + \frac{M}{\lambda}[e^{-\lambda T} - 1]$

Now the deterioration cost for the interval [0,T] = C x (the number of deteriorated items)

$$= C\left[\frac{M}{(\theta-\lambda)}[e^{(\theta-\lambda)T} - 1] + \frac{M}{\lambda}[e^{-\lambda T} - 1]\right] = CMe^{-\lambda T}\left[\frac{1}{(\theta-\lambda)}[e^{\theta T} - e^{\lambda T}] + \frac{1}{\lambda}[1 - e^{-\lambda T}]\right] \tag{4}$$

(3). Total Holding Cost

The total inventory holding cost for the interval [0,T] is = $\int_0^T (g + at)Q(t)dt$

$$=\int_0^T (g+at)\left[\frac{M}{(\theta-\lambda)}[e^{(\theta-\lambda)T-\theta t}\text{-}e^{-\lambda t}\ ]\right]dt$$

$$=\frac{Me^{-\lambda T}}{\theta\lambda(\theta-\lambda)}\left[(g+at)(\theta-\lambda)-g(\theta e^{\lambda T}-\lambda e^{\theta T)}+\frac{a}{\theta\lambda}[(\theta^2-\lambda^2)-(\theta^2 e^{\lambda T}-\lambda^2 e^{\theta t)}]\right] \tag{5}$$

Now, we find the total variable cost per unit time

= ordering cost + deterioration cost + inventory holding cost

$$=\frac{P}{T}+\frac{CMe^{-\lambda T}}{T}\left[\frac{1}{(\theta-\lambda)}[e^{\theta T}-e^{\lambda T}]+\frac{1}{\lambda}[1-e^{-\lambda T}]\right]+$$

$$\frac{Me^{-\lambda T}}{\theta\lambda(\theta-\lambda)T}\left[(g+at)(\theta-\lambda)-g(\theta e^{\lambda T}-\lambda e^{\theta T)}+\frac{a}{\theta\lambda}[(\theta^2-\lambda^2)-(\theta^2 e^{\lambda T}-\lambda^2 e^{\theta t)}]\right] \tag{6}$$

To minimize the total cost (TC), the conditions are $\frac{\partial TC(T)}{\partial T}=0$ and $\frac{\partial^2 TC(T)}{\partial T^2}>0$.

Now $\frac{\partial TC(T)}{\partial T}=0$ gives the following equation in T:

$$\frac{\partial TC(T)}{\partial T}=\frac{Me^{-\lambda T}}{\theta^2\lambda T(\theta-\lambda)}[a\theta(\theta-\lambda)-g\theta^2\lambda(e^{\lambda T}-e^{\theta T})a\theta(\lambda e^{\theta T}-\theta e^{\lambda T})-\theta\lambda(\theta-\lambda)(g+aT)$$

$$+g\theta\lambda(\theta e^{\lambda T}-\lambda e^{\theta T})-a(\theta^2-\lambda^2-\theta^2 e^{\lambda T}+\lambda^2 e^{\theta T})]$$

$$+\frac{CMe^{-\lambda T}}{(\theta-\lambda)T}[\theta e^{\theta T}-\lambda e^{\lambda T})-\lambda(e^{\theta T}-e^{-\lambda T})-\theta+\lambda]-\frac{TC}{T} \tag{7}$$

Provided it satisfies the condition $\frac{\partial^2 TC(T)}{\partial T^2}>0$

Now,

$$\frac{\partial^2 TC(T)}{\partial T^2}=\frac{Me^{-\lambda T}}{T^2}[\theta(\theta-\lambda)(g\lambda^2 T+aT^2\lambda-2a\lambda T)\theta^2\lambda(e^{\lambda T}-e^{\theta T})(2\lambda gT-aT+2g)+\theta(\lambda e^{\theta T}-\theta e^{\lambda T})$$

$$(\lambda T-2a+2g\lambda)+a(\lambda T+2)(\theta^2-\lambda^2-\theta^2 e^{\lambda T}+\lambda^2 e^{\theta T})-gT\theta^2\lambda(\lambda e^{\lambda T}-\theta e^{\theta T})+2\theta\lambda(\theta-\lambda)(g+aT)]$$

$$+\frac{CMe^{-\lambda T}}{(\theta-\lambda)T^2}[(\theta-\lambda)Te^{\theta T}+\lambda T+2)+2\lambda(e^{\theta T}-e^{-\lambda T})-2\lambda^2 Te^{-\lambda T}-2\theta e^{\theta T}+2\lambda e^{\lambda T}]+\frac{2(TC)}{T^2}$$

Now, we can see for $\frac{\partial^2 TC(T)}{\partial T^2} > 0$ the optimal value of T obtained from (7), it means that (6) has a unique solution

Numerical Example

Following numerical example will illustrate the model

The values of the various parameters in proper units can be taken as follows:

P= 550, M = 300, λ = 0.02, ,θ = 0.6, a = 0.25 and C = 1

Put the values of these parameters in equation (6) and solve, we have T* = 1.23328. Now put this value in equation (6) and (3) , we obtain the minimum total cost per unit time TC*= 711. 42 and the economic order quantity Q_0 = 512. 385.

Conclusion

In the present study an inventory model is presented, which locates the optimal ordering quantity of inventory system due to an exponential decaying demand rate and linear time dependent holding cost. The deteriorating items (like foods, flowers fashion items, computer chips, etc.) have fixed life which is lost with time during the end of season. Also the storage period for which the demand, deterioration, and holding cost depend on the time where the shortages are not allowed. This model is solved so that the total inventory cost is to be minimized. A numerical example is given to illustrate the model. In the future study, the model can be extended for deteriorating items in many situations such as for items having linear and quadratic increasing demand, stock-in and stock-out dependent demand, price dependent demand or power demand.

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